

Critical dynamics and global persistence exponent on Taiwan financial market

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Abstract

We investigated the critical dynamics on the daily Taiwan stock exchange index (TSE) from 1971 to 2005, and the 5-min intraday data from 1996 to 2005. A global persistence exponent θ_p was defined for non-equilibrium critical phenomena [1, 2], and describing dynamic behavior in an economic index [3].

In recent numerical analysis studies of literatures, it is illustrated that the persistence probability has a universal scaling form $P(t) \sim t^{-\theta_p}$ [4]. In this work, we analyzed persistence properties of universal scaling behavior on Taiwan financial market, and also calculated the global persistence exponent θ_p . We found our analytical results in good agreement with the same universality.

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I. INTRODUCTION

Problems in economy and finance have attracted the interest of statistical physicists all over the world. Using the tools developed for statistical physics, like phase transitions, critical exponents, mean field approximations, renormalization group [5], persistence probability [6, 7, 8].

In recent years the detrended fluctuation analysis (DFA) method [9, 10, 11, 12, 13, 14] has become a widely used technique for the determination of (mono-) fractal scaling properties and the detection of long-range correlations in noisy, nonstationary time series [11]. In many of non-equilibrium systems, the persistence has been found to decay as a power-law at time series, $P(t) \sim t^{-\theta_p}$. Hurst exponent and persistence exponent in these financial time series are investigated in numerical and analytical [8].

We calculated the experimental data with the daily Taiwan stock exchange index (TSE) from 1971 to 2005, and the 5-min intraday data from 1996 to 2005. In this work, we analyzed persistence properties of universal scaling behavior on Taiwan financial market.

II. METHOD

We consider a set of data recorded the daily Taiwan stock exchange index (TSE) from 1971 to 2005. Let $Y_i(t)$ be the stock index at discrete times i , $i = 1, 2, \dots, t_n - 1$. The final transaction time is denoted by t_n . Then, the log-return price is defined as

$$r_i(t) = \ln Y_i(t + \Delta t) - \ln Y_i(t), \quad (1)$$

where Δt is time interval. In this paper, we analyzed daily data; $\Delta t = 1$ day.

Let us denote the value of the TSE at a certain time t' as $y(t')$. $P_+(t)$ is the probability that the value of the TSE has never gone down to the value $y(t')$ in time t , $y(t' + N\Delta t) > y(t')$ for $N = 1, 2, \dots, t$. i.e., $P_-(t)$ is the probability that the value of the TSE has never gone up to the value $y(t')$ in time t , $y(t' + N\Delta t) < y(t')$ for $N = 1, 2, \dots, t$. The persistence probability $P(t)$ is $[P_+(t) + P_-(t)]/2$.

The persistence probability has a power-law behavior

$$P(t) \sim t^{-\theta_p}. \quad (2)$$

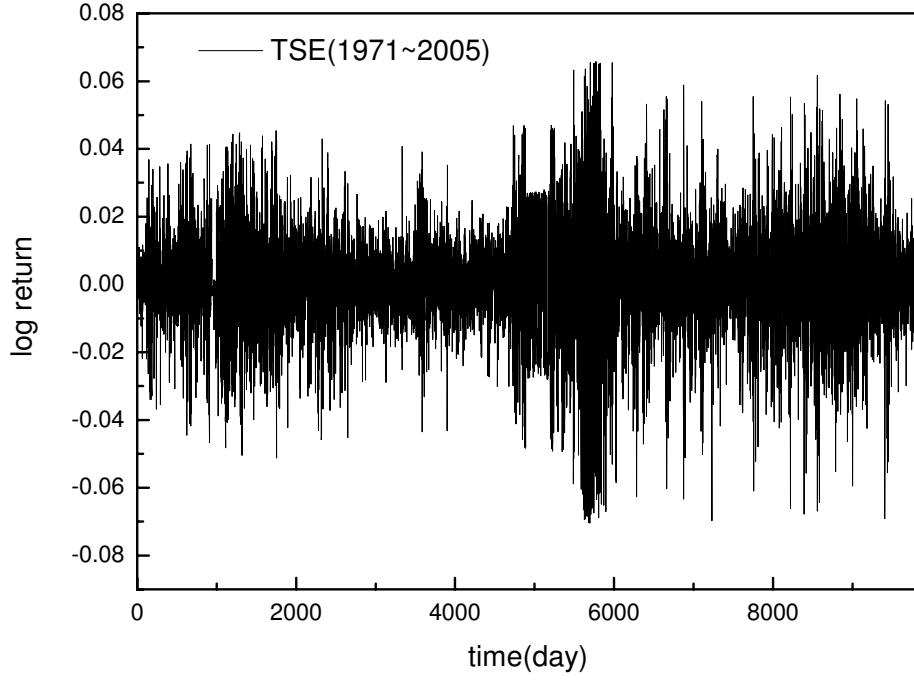


FIG. 1: Plot of the log-return price $r_i(t)$ vs. t on the daily Taiwan stock exchange index (TSE) from 1971 to 2005.

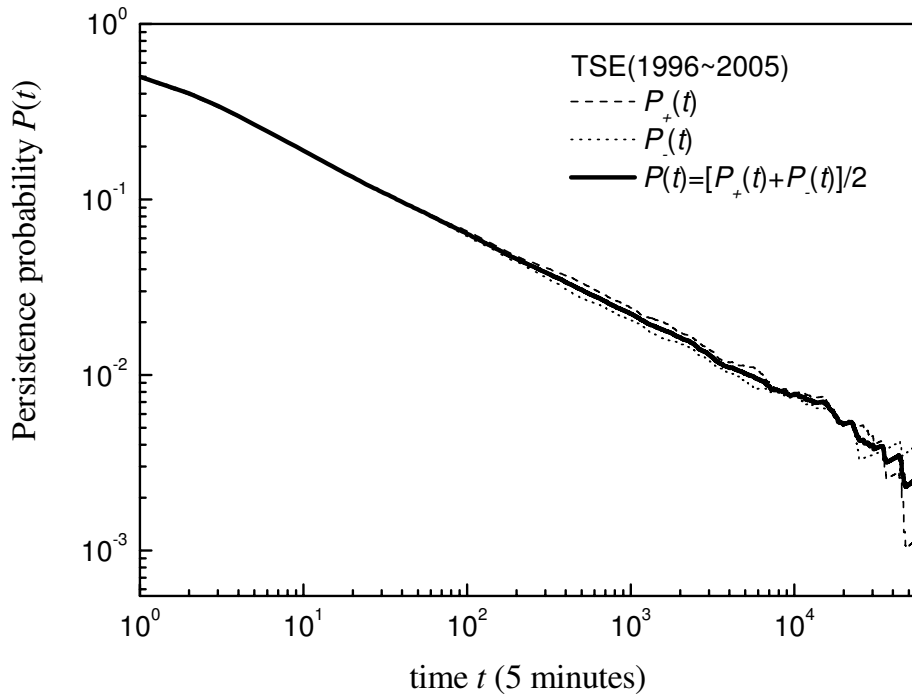


FIG. 2: Persistence probability with 5 minutes data of the daily Taiwan stock price index (TSE) from 1996 to 2005.

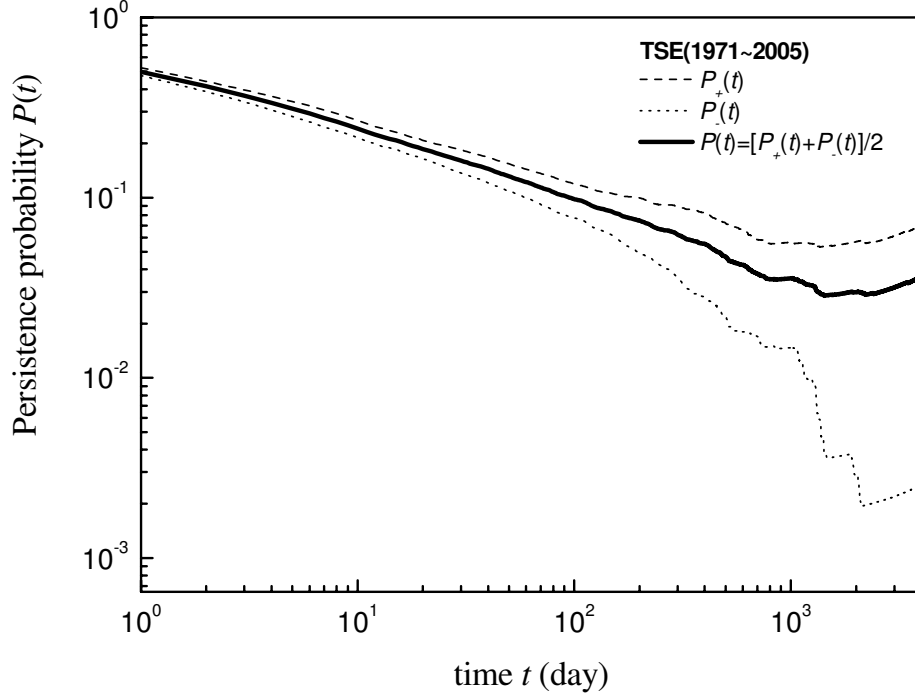


FIG. 3: Persistence probability with 5 minutes data of the daily Taiwan stock price index (TSE) from 1996 to 2005.

For the cumulative time series of the log-return price variables is defined as

$$X(i) = \sum_{k=1}^i (r_k - \bar{r}), \quad (3)$$

where \bar{r} is the average value of log-return price. $X(i)$ is divided into Ns disjoint segments of length s . $p_\nu(i) = a_i + b_i t$, a_i, b_i is constant. Since the length N of the series is often not a multiple of the considered

$$X_s(i) = X(i) - p_\nu(i). \quad (4)$$

The generalized q th-order price-price correlation function is defined as

$$G_q(t) = \langle |Y(t_0 + t) - Y(t_0)|^q \rangle^{1/q}, \quad (5)$$

where $Y(t)$ is the stock price and the average is over all the initial times t_0 . $G_q(t)$ has a power-law behavior

$$G_q(t) \sim t^{H_q}. \quad (6)$$

where H_q is called the generalized Hurst exponent.

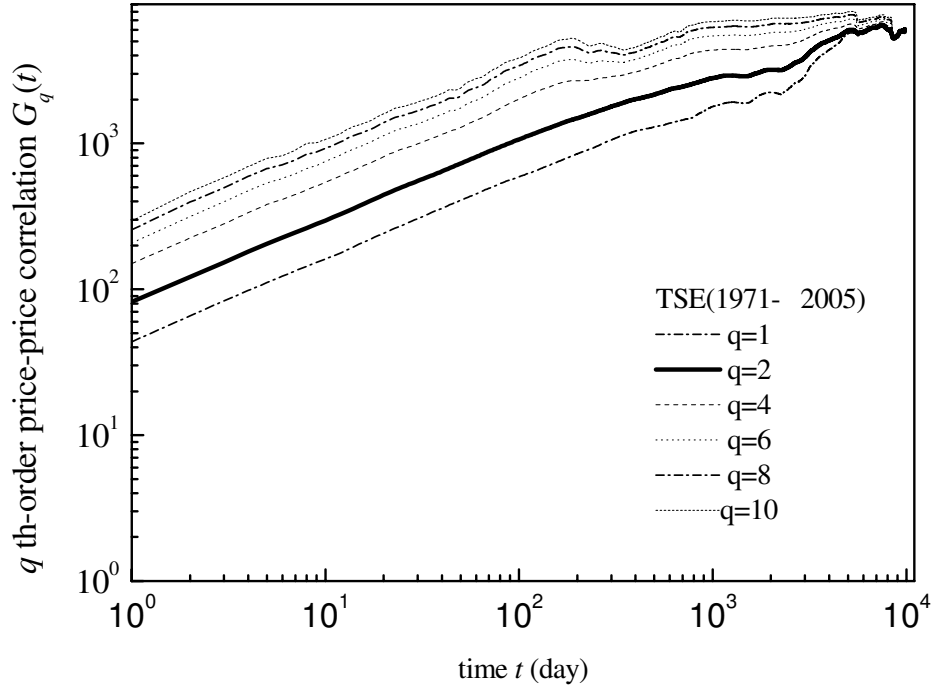


FIG. 4: Log-log plot of the generalized price-price correlation function $G_q(t)$ vs. t corresponding to the daily Taiwan stock exchange index (TSE) from 1971 to 2005.

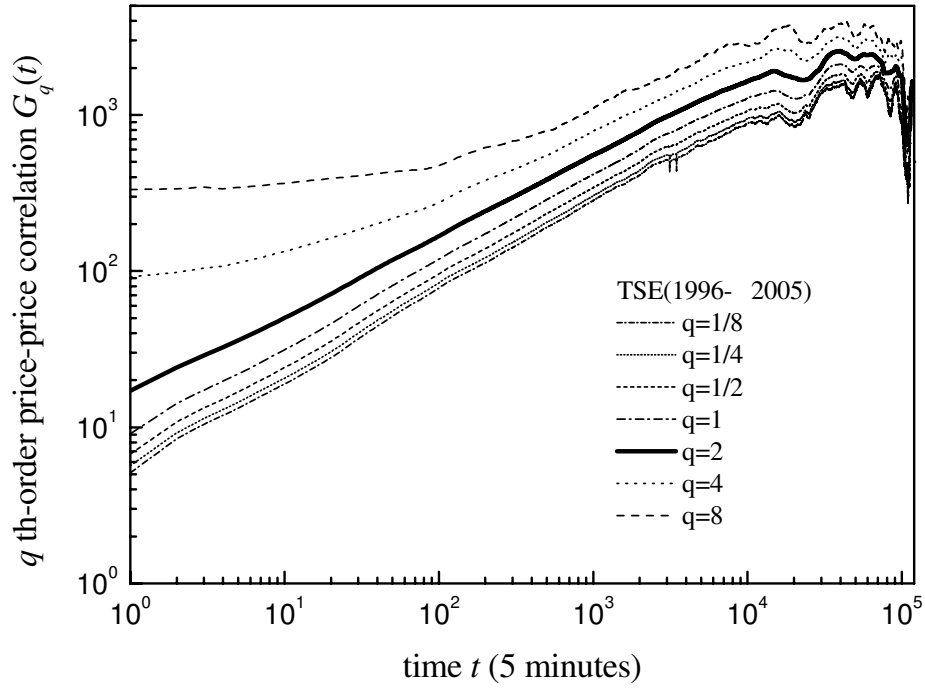


FIG. 5: Log-log plot of the generalized price-price correlation function $G_q(t)$ vs. t corresponding to 5 minutes data of the daily Taiwan stock price index (TSE) from 1996 to 2005.

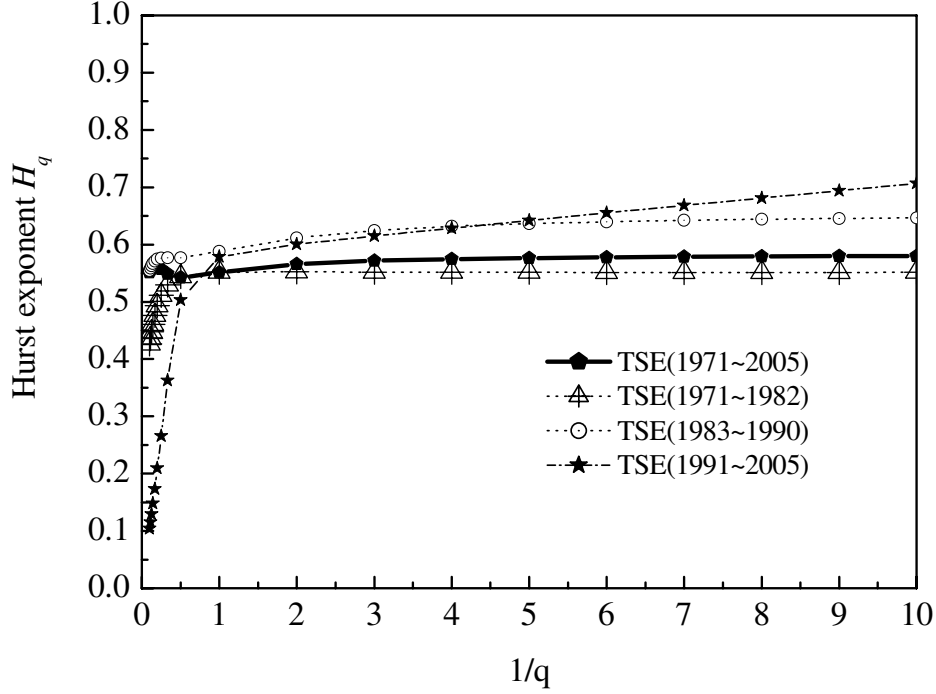


FIG. 6: Plot of the generalized Hurst exponent H_q vs. $1/q$ on differ daily Taiwan stock exchange indices.

The generalized fluctuation function is defined as

$$F_s^2(\nu) = \left[\frac{1}{s} \sum_{j=1}^s X_s \right]^2. \quad (7)$$

The generalized q th-order fluctuation function is defined as

$$F_q(s) = \left[\frac{1}{2N} \sum_{\nu=1}^{2N_s} F_s^2(\nu)^{q/2} \right]^{1/q}. \quad (8)$$

By construction, since we use a linear fit for simplicity, $F_q(s)$ is defined for $s \geq 3$. The scaling form of the correlation function $F_q(s) \sim H_q$ provides the family of generalized Hurst exponents H_q . For reasons that will become clearer very shortly we also introduce the dimensionless fluctuation function is defined as

$$f_q(s) = \frac{[F_s^2(\nu)^{q/2}]^{1/2}}{[\frac{1}{N} \sum_k^i (r_k - \bar{r})^2]^{1/2}}. \quad (9)$$

Let us denote the value of the TSE at a certain time t' as $Y(t')$. We calculate the pdf $P(Z_1)$ of the index changes

$$Z_1(t') = Y(t' + \Delta t) - Y(t'). \quad (10)$$

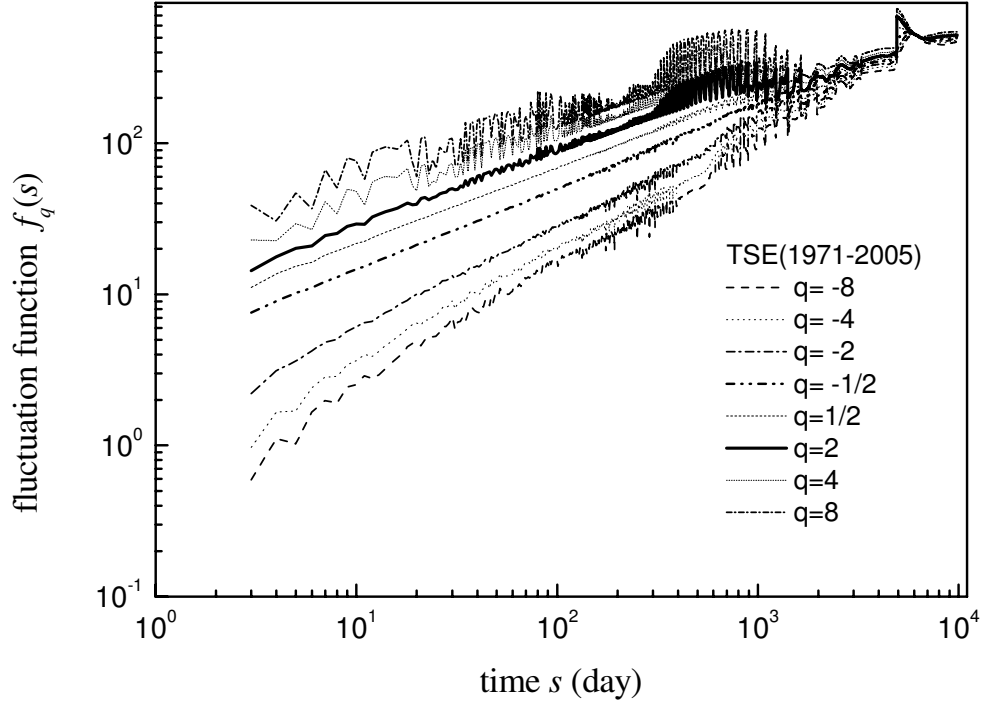


FIG. 7: Log-log plot of the normalized fluctuation function $f_q(s)$ vs. s corresponding to the daily Taiwan stock exchange index (TSE) from 1971 to 2005.

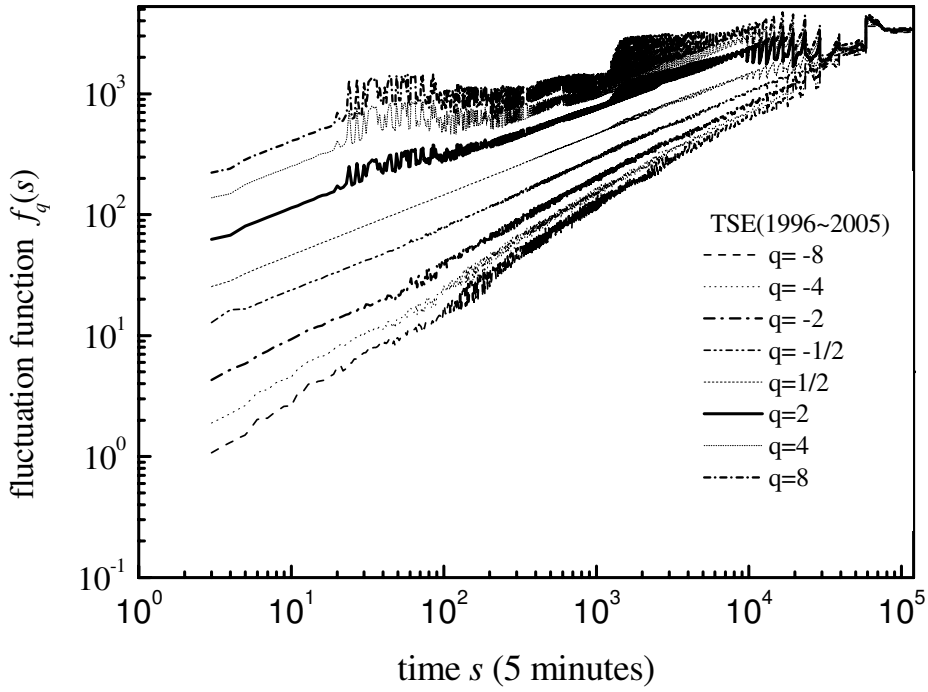


FIG. 8: Log-log plot of the normalized fluctuation function $f_q(s)$ vs. s corresponding to 5 minutes data of the daily Taiwan stock price index (TSE) from 1996 to 2005.

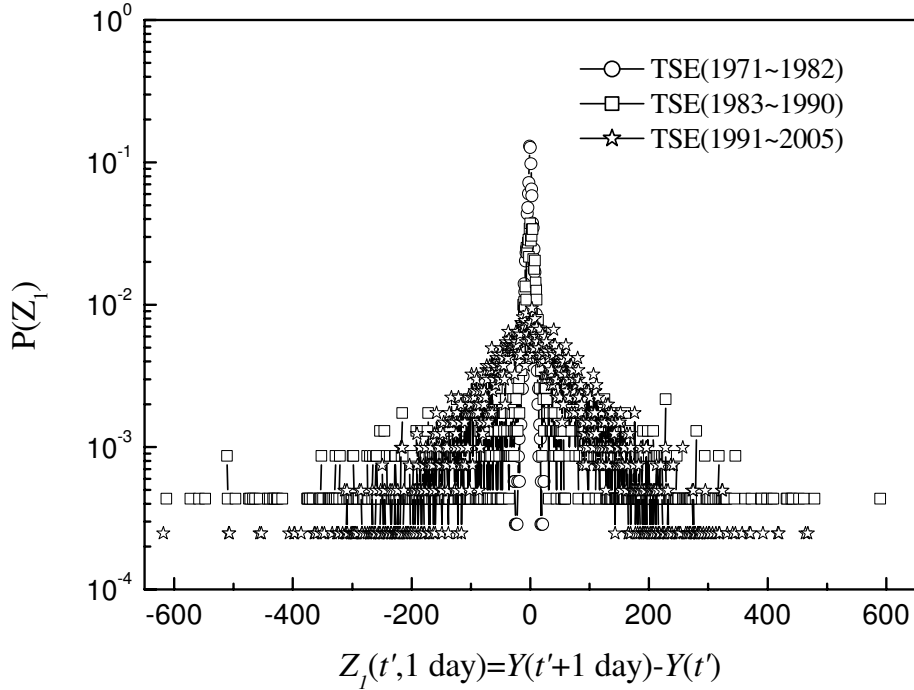


FIG. 9: Plot of the $P(Z_1)$ vs. $Z_1(t')$ differ time data on the daily Taiwan stock exchange index (TSE).

III. DISCUSS AND RESULTS

A "persistence exponent" θ_p is defined for non-equilibrium critical phenomena. Based on large amounts of data compiled in past years, especially those records in minutes or seconds, it becomes possible to perform relatively accurate analysis and to study the fine structure of the dynamics. In Fig. 2 and 3, we observe the persistence probability that at least up to least up to 250. The slope of persistence probability $P(t)$ estimation from the initial time to 200.

The price evolution is multifractal if the exponent hierarchy H_q varies with q , otherwise is fractal in the theory of surface dynamical scaling referred to as multiaffine and self-affine, respectively. In particular, for $q = 2$, we recover the fractional Brownian motion case described by the well-known Hurst exponent, $0 < H_2 < 1$. The bridge between these two analyses is provided by the second-order Hurst exponent H_2 associated with the correlation function of the stock price, which has been shown to be simply related to the persistence exponent through $H_2 = 1 - \theta_p$.

We note that this relation holds for any zero-mean process (not necessarily Gaussian

TABLE I: Compare H_2 , θ_p value on the daily Taiwan stock exchange index (TSE).

Time	H_2	θ_p
1971 - 1982 (daily)	0.54	0.41
1983 - 1990 (daily)	0.58	0.34
1991 - 2005 (daily)	0.50	0.42
1971 - 2005 (daily)	0.54	0.46
1996 - 2005 (5 minutes)	0.52	0.46

[15, 16])that satisfies requirements above.

IV. CONCLUSION

We analyze the daily Taiwan stock exchange index (TSE) from 1971 to 2005 and the 5-min intraday data from 1996 to 2005. The persistence exponent θ_p associated with the power-law decay of the average probability.

Our studies base on the persistence probability analysis of the critical behavior in an economic index, and the numerical estimation of the persistence exponent θ_p with Hurst exponent H_2 .

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